Handout for Two Rods

SOLUTION

The fastest approach we know is to perform six binary searches.

- Using entire rows/columns as the query rectangle, the top and bottom rows, and leftmost and rightmost columns containing any portion of a rod can be found using 4 binary searches, or $4\lceil \lg N \rceil$ calls to `rect`.
- We now have the smallest rectangle containing all of both rods. By checking the corners of this rectangle (4 calls to `rect`, each with a 1 by 1 query rectangle), we can determine the general form of the structure, as in the examples of the figure and their rotations.
- Finally the solution can be found in one or two more binary searches depending on the case.

3 corners intersect rods | 2 opposite corners intersect rods | 2 adjacent corners intersect rods | no corners intersect rods

This leads to a $6\lceil \lg N \rceil + 4$ solution. The maximum value of $\lceil \lg N \rceil$ in the test data is 14, so we have a solution that takes at most 88 calls to `rect`. With care, this can be reduced to $6\lceil \lg N \rceil + 1$. Notice that the queries we ask of the data have only two possible answers. As there are $N^4(N\text{-}1)^2/4$ possible placements of the rods, $\lceil \lg (N^4(N\text{-}1)^2) \rceil -2 \approx \lceil 6 \lg N \rceil -2$ calls are necessary, on average, for any algorithm. We do not claim our testing is exhaustive, so we simply take this as a worst case lower bound. We implemented two versions of the general approach suggested.

There are a number of variations on this approach. For example, one could try to finding the bounding rectangle more quickly when it is large. Approaches of this type tend to double the number of calls to `rect` and will lead to full marks on the 4 small cases and 3 marks on each of the larger cases.

The most naïve approach involves scanning individual cells to find some portion of a rod, then looking around for the rest of the rod. This can clearly lead to an N^2 solution, or even slightly worse if one is careless. The approach receives full marks in the four cases of size 10. Another exhaustive approach involves taking entire rows (or columns) as query rectangles to find the bounding rectangle, then applying a similar approach to find the rods inside this rectangle. This will require $O(N)$ calls, though the constant will vary with details of the implementation. Depending on the details of the implementation, such an approach could also gain credit in several of the larger examples in which the rods are small and near a corner.

Testing Data Description for RODS

No	Grid Size, N	Solution 1	Solution 2	$6\lceil \lg N \rceil + 4$
1	10	14	15	28
2	1000	51	51	64
3	5000	77	77	82
4	7000	75	76	82
5	10000	80	79	88
6	10	17	18	28
7	1000	50	51	64
8	5000	69	68	82
9	7000	77	76	82
10	10000	79	77	89
11	10	16	15	28
12	1000	62	63	64
13	5000	73	71	82
14	7000	79	76	82
15	10000	79	77	88
16	10	16	15	28
17	1000	52	51	64
18	5000	56	61	82
19	7000	65	64	82
20	10000	70	70	88