

UTOPIA DIVIDED

Solution

The two dimensional problem can be solved by two separate one dimensional subproblems. The one dimensional problem is: Given N code numbers and a sequence of N region signs (each of which is + or -), produce a sequence of N signed code values $\{x_i\}$ so that the sign of $\sum_{i \leq k} x_i$ matches the i^{th} region sign.

We do this by first sorting the N input code numbers into increasing order, and then assigning alternating signs to them so that $|x_i| > |x_{i+1}|$, though $x_i > 0$ iff $x_{i+1} < 0$. The key observation is that if we have used a segment (x_k, \dots, x_m) of the data, then $0 \leq \left| \sum_{k \leq i \leq m} x_i \right| \leq |x_m|$, and the sign of the sum matches that of x_m . If the next code is to reverse the sign, we simply use x_{m+1} , which guarantees the invariant of the last sentence still holds. A similar argument guarantees that using x_{k-1} retains the sign of the sum.

The next important issue is where to start. Recall that keeping the sign of the sum the same requires taking a new value from the right. So count the number of sign changes in the input sequence of region signs. If this value is c , give x_c the first input sign, thus determining the alternation. The output can now be produced in a fairly straightforward manner.

Hence there is an $O(N \lg N)$ solution that can be coded (if not discovered) quite easily. It is also quite reasonable to solve the problem by backtracking. This leads to an acceptable solution on smaller test cases (up to 8, or with care to 9 or even 10).

No	Size, N	Description
1	N = 4	Example 2
2	N = 10	Key value : 1~20: Circular plane sweep
3	N = 10	Key value : Starting from 20, step 1 or 2, scrambled
4	N = 30	Key value : Starting from 30, step 1 or 2, scrambled
5	N = 30	Key value : Starting from 200, step 1 to 3, scrambled
6	N = 50	Key value : Starting from 1, step 1 to 3, scrambled
7	N = 50	Key value : Starting from 150, step 1 to 3, scrambled
8	N = 50	Key value : Starting from 300, step 1 to 4, scrambled Visit plane 1 except last visit. On last, visit plane 3
9	N = 100	Key value : Starting from 1000, step 1 to 4, scrambled. Circular plane visit
10	N = 500	Key value : 1, 3, 5, ..., 1999. Circular plane visit
11	N = 700	Key value : Starting from 1, step 1 to 5, scrambled. Visit plane 1, 2 only
12	N = 1000	Key value : Starting from 4000, step 1 to 5, scrambled
13	N = 1500	Key value : Starting from 9000, step 1 to 5, scrambled
14	N = 2000	Key value : Starting from 15000, step 1 to 5, scrambled
15	N = 2500	Key value : Starting from 30000, step 1 to 5, scrambled
16	N = 3000	Key value : Starting from 1, step 1 to 7, scrambled. Visit plane 1, 4 only
17	N = 3500	Key value : Starting from 20000, step 1 to 7, scrambled
18	N = 4000	Key value : Starting from 30000, step 1 to 7, scrambled
19	N = 4500	Key value : Starting from 50000, step 1 to 7, scrambled
20	N = 5000	Key value : Starting from 60000, step 1 to 7, scrambled
21	N = 6000	Key value : Starting from 1, step 1 to 9, scrambled. Visit plane 2, 4 only
22	N = 7000	Key value : Starting from 40000, step 1 to 7, scrambled
23	N = 8000	Key value : Starting from 50000, step 1 to 5, scrambled
24	N = 9000	Key value : Starting from 30000, step 1 to 6, scrambled
25	N = 10000	Key value : Starting from 80000~99999, scrambled