XOR

Solution

In addition to pixels, we examine *grid points*, which are intersection points of two grid lines. An *image corner* is grid point adjacent to an odd number of black pixels. This way, an image is all white if and only if there are no image corners.

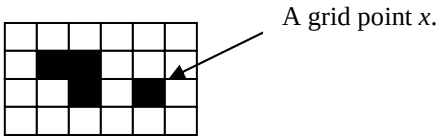

Figure 1. An example image.

For example, in Figure 1 the grid point *x* is adjacent to four pixels, of which 1 is black and 3 are white. This way, *x* is also an image corner. In total, there are 10 image corners in the image in Figure 1.

We may now try to find a solution by working from the input image backward to an all white image by doing XOR operations. It can be shown that if there are black pixels in the image, then it is always possible to find at least three such image corners that they are corners of the same rectangle. Now we use the following algorithm.

Algorithm 1. Two-optimal XOR

1. If there are black pixels in the image, find three such image corners that they are corners of the same rectangle, and use the respective XOR call. Go to 1.
2. Halt.

It can be shown that repeating the execution of Step 1 will eventually remove all black pixels and, in this way, create a solution. It can also be shown that we can reduce the number of image corners by at most four with a single XOR call, and the above algorithm always reduces the number of image corners by at least two. Therefore, the above algorithm always uses at most twice as many XOR calls than the best possible solution (and we say that Algorithm 1 is *2-optimal*).

We can produce variations of Algorithm 1 by different ways of choosing the image corners, and they may produce different numbers of XOR calls.

There are also other approaches to solve this problem. As an extremely naïve solution, one may do one XOR call for each black pixel in the image. Then, you will do as many XOR calls as there are black pixels in the image. As an improvement, you may also try to identify all-black rectangles, which may considerably reduce the number of XOR calls from the previous naïve solution.